Some Basic Image Manipulations

Preamble

Preamble Utilities for this worksheet

$\text{Clear}(R, G, B, IMG) = 1$ $I := [R\ G\ B]$ $R := IMG_1$ $G := IMG_2$ $B := IMG_3$

$m := \text{rows}(R)$ $n := \text{cols}(R)$ $r := [1..m]$ $c := [1..n]$ $Z := \text{eval}(\text{matrix}(m, n))$

$$XY(dummy) := \left[\left[\left[\begin{matrix} \text{"image:\{I\}"} \\ \text{stack}(0, 0, \text{rows}(R), \text{cols}(R)) \\ \text{"transparent"} \\ \text{"line"} \\ 1 \end{matrix}\right]\right]\right]$$

Basics

Tero Matrix

Basic examples $IMG := Tero$

We can use an image component for show images, but in this worksheet we use XYPlot's components.

$[R\ G\ B]$

$$\left[\begin{matrix} IMG01'XAxis'Min := 0 & IMG01'XAxis'Max := m \\ IMG01'YAxis'Min := 0 & IMG01'YAxis'Max := n \end{matrix}\right]$$

$XY(I := [R\ G\ B])$

$XY(I := [R\ Z\ Z])$

$XY(I := [Z\ G\ Z])$

$XY(I := [Z\ Z\ B])$

$XY(I := 255 - [R\ G\ B])$

$XY(I := 255 - [R\ Z\ Z])$

$XY(I := 255 - [Z\ G\ Z])$

$XY(I := 255 - [Z\ Z\ B])$

Grayscale: The image is defined by only one matrix. We can define it as the mean of the three colors, but looks a little better this ponderation:

$$GS0 := \frac{1}{3}\cdot(R+G+B)$$

$$GS := \text{eval}\left(\frac{127}{255}\cdot R + \frac{100}{255}\cdot G + \frac{28}{255}\cdot B\right)$$

XY(I := GS0)

XY(I := GS)

We can use a GS matrix for adding some color schema

$$MC(GS, c) := \left|\text{eval}\left(\frac{1}{255}\cdot\left[c_1\cdot GS\ c_2\cdot GS\ c_3\cdot GS\right]\right)\right.$$

$$wheat := [245\ 222\ 179]$$
$$yellow := [255\ 255\ 0]$$
$$violet := [127\ 0\ 255]$$

XY(I := MC(GS, wheat))

XY(I := MC(GS, yellow))

XY(I := MC(GS, violet))

We also can apply some functions to each point in the image

$$f(x) := \left|\text{eval}\left(\overrightarrow{100\cdot\log10(x+1)}\right)\right.$$

$$g(x, y) := \left|\text{eval}\left(\overrightarrow{255\cdot e^{\frac{x+0.25\cdot y}{1.25\cdot 255}}}\right)\right.$$

$$h(x, y) := \left|\text{eval}\left(\frac{\overrightarrow{\sqrt{x^2+y^2}}}{\sqrt{2\cdot 255^2}}\right)\right.$$

XY(I := [f(R) G B])

XY(I := [R g(B, G) B])

XY(I := [R G h(B, R)])

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svd image

SVD Decomposition

In the singular value decomposition, the matrices U,S and V can be interpreted as the weigted components of a series sum for the matrix A. If the first elements in the spectrum of the weights w are bigger than the rest, then the original matrix can be weel approximate with some few elements of the serie sum. We can apply this to an imagge.

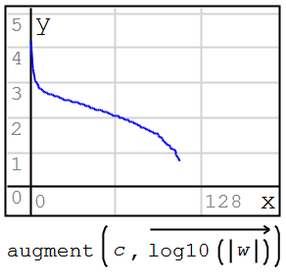
$$MSVD(A) := \left|\begin{bmatrix} S \\ U \\ V' \end{bmatrix} := \text{dn\_LinAlgSVD}(A)\right.$$
$$\left.\begin{bmatrix} -U & S & -V' \end{bmatrix}\right.$$

$$[U\ S\ V'] := MSVD(GS)$$

$$\left|\begin{matrix} r := [1..m] \\ c := [1..n] \end{matrix}\right.$$

$$w_c := S_{c\,c}$$

$$\sum\left(\overrightarrow{w > 200}\right) = 42$$



$$\text{augment}\left(c, \overrightarrow{\log10(|w|)}\right)$$

$$img(k) := \left|\text{eval}\left(U_{r\,k}\cdot S_{k\,k}\cdot V'_{k\,c}\right)\right.$$



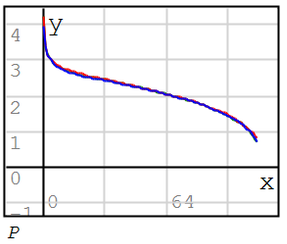

XY(I := img([1..22])) XY(I := img([1..44])) XY(I := img([1..66])) XY(img([1..88]))

We can apply the same to each color.

$$\begin{bmatrix} UB & SB & V'B \end{bmatrix} := MSVD(B)$$
$$\begin{bmatrix} UR & SR & V'R \end{bmatrix} := MSVD(R)$$
$$\begin{bmatrix} UG & SG & V'G \end{bmatrix} := MSVD(G)$$

$$wr_{c} := SR_{c\,c}$$
$$wg_{c} := SG_{c\,c}$$
$$wb_{c} := SB_{c\,c}$$

$$P := \begin{cases} \text{augment}\left(c, \overrightarrow{\log10(|wb|)}\right) \\ \text{augment}\left(c, \overrightarrow{\log10(|wr|)}\right) \\ \text{augment}\left(c, \overrightarrow{\log10(|wg|)}\right) \end{cases}$$



$$\sum\left(\overrightarrow{wr > 200}\right) = 42 \qquad \sum\left(\overrightarrow{wg > 200}\right) = 43 \qquad \sum\left(\overrightarrow{wb > 200}\right) = 41$$

$$img3(k) := \left| \text{eval}\left(\begin{bmatrix} UR_{r\,k} \cdot SR_{k\,k} \cdot V'R_{k\,c} & UG_{r\,k} \cdot SG_{k\,k} \cdot V'G_{k\,c} & UB_{r\,k} \cdot SB_{k\,k} \cdot V'B_{k\,c} \end{bmatrix}\right)\right.$$




XY(I := img3([1..12])) XY(I := img3([1..24])) XY(I := img3([1..36])) XY(img3([1..48]))

Images are Functions

Images are functions

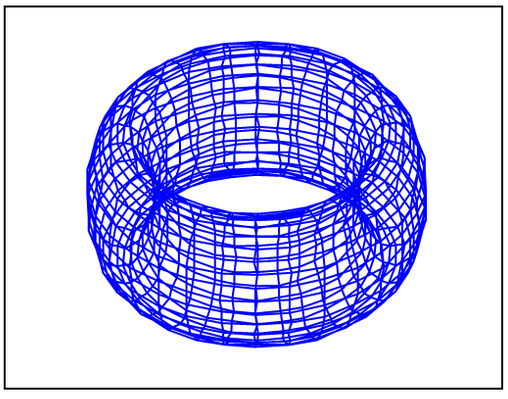
The image component take 3 matrices with the same dimensions as the RGB colors. This values represents an image with 31x31 pixels ... and a torus.

$$CM(\varphi) := \left| \begin{aligned} &f\#(x, y) := \text{eval}(\varphi) \\ &\text{CreateMesh}(f\#, 1, m, 1, n, m-1, n-1) \end{aligned}\right.$$

TorusRGB matrix

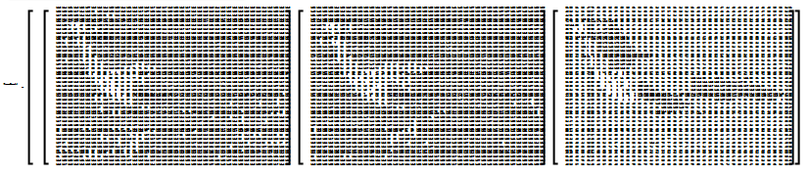
$$\left| \begin{aligned} &IMG := TorusRGB \\ &f(x, y) := \begin{bmatrix} R_{x\,y} \\ G_{x\,y} \\ B_{x\,y} \end{bmatrix} \\ &\gamma 2 := \begin{bmatrix} 0.866 & -0.433 \\ 0.5 & 0.75 \\ 0 & 0.5 \end{bmatrix} \\ &\Pi := CM(f(x, y)) \cdot \gamma 2 \end{aligned}\right.$$

$$[m \;\; n] = [31 \;\; 31]$$



But that trick works only for some few images. Usually, we can represent the 3 colors for separate for "see" the function under the image

TeroSmall matrix



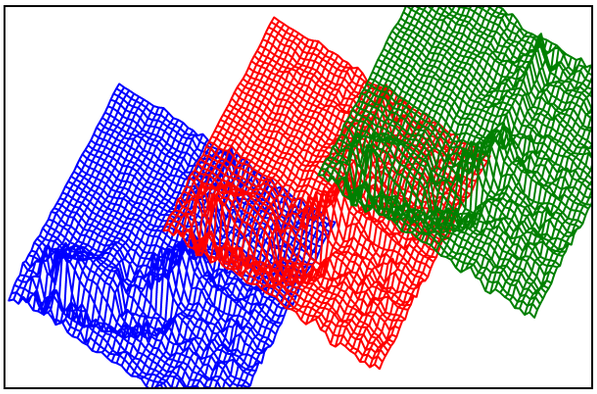
$IMG := TeroSmall$

$sc := \mathrm{diag}\left(\begin{bmatrix} 15 & 15 & 1 \end{bmatrix}^{T}\right)$

$$\Pi := \begin{cases} CM\left(\begin{bmatrix} x \\ y \\ B \\ x & y \end{bmatrix}\right) \cdot sc \cdot \gamma 2 \\ CM\left(\begin{bmatrix} x \\ y \\ R \\ x & y \end{bmatrix} + 20\right) \cdot sc \cdot \gamma 2 \\ CM\left(\begin{bmatrix} x \\ y \\ G \\ x & y \end{bmatrix} + 40\right) \cdot sc \cdot \gamma 2 \end{cases}$$

$[R \; G \; B]$

$[m \; n] = [45 \; 40]$



Alvaro

$\mathrm{appVersion}(4) = "1.2.9018.0"$