How to find the

1. Two Intersection Points **(AA and CC)** of a defined **CENTER** (x,y) of a slip circle with the **SLOPE Profile** .
2. Number of **SLICES** for a given **width** of slices.

Based on Example 12.2 (Page 296) - Soil Mechanics : Principles and Practice by G.E. Barnes

$t_0 := \text{time}(0)$

appVersion(4) = "1.0.8348.30405" appVersion(−4) = "1.0.8348.30405"

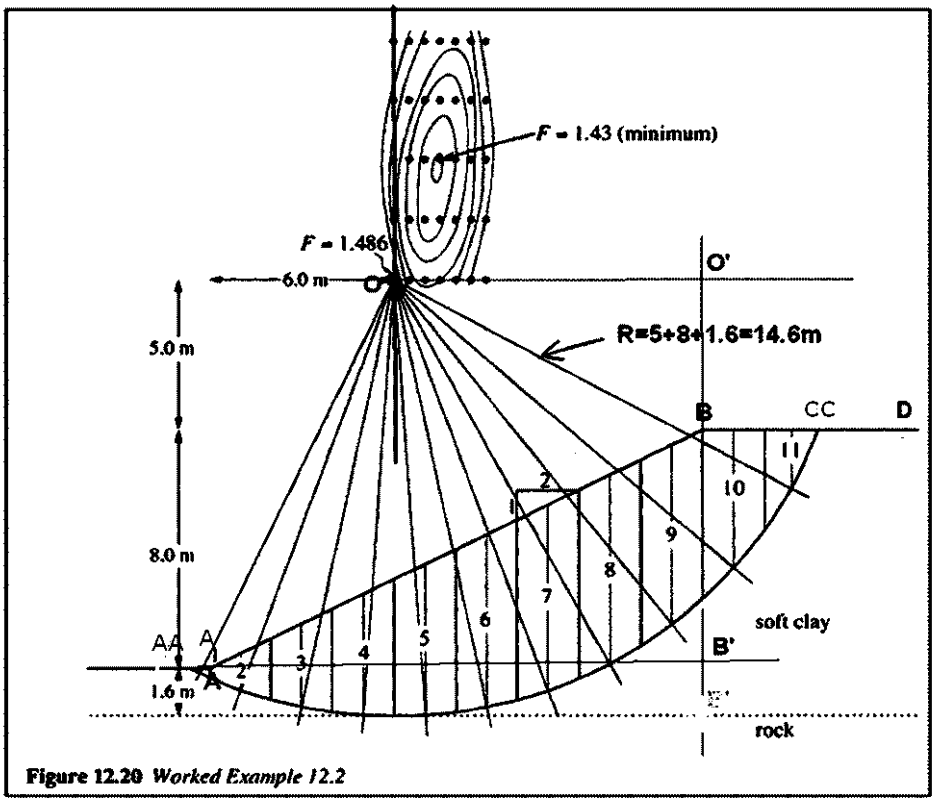

Figure 12.20 *Worked Example 12.2*

From Textbook
A slope is to be cut into a soft clay with undrained shear strength of 30 kN/m2 and unit weight of 18 kN/m3• The slope is 8.0 m high and its inclination is 2: 1 (horizontal:vertical). Determine the factor of safety for the trial circle shown on **Figure 12.20.**

Slip Circle **tangential** to the **ROCK** Layer at depth of **1.6 m** below the toe.

METHOD of Analysis :
Note : ***Textbook method using eqn 12.9*** **is** ***approximate.***
Instead, an accurate method of finding Strip Areas (Program 21) and Full Curve Length (Program 22) is used

Slope Geometry

$(H_{bund} := 8\ \text{m}) = \text{"Height of Bund"}$
$(\beta_{slope} := \frac{1}{2}) = \text{"Slope of line AB with Horizontal"}$
$(H_{hz} := 6\ \text{m}) = \text{"Horizontal distance from A to Center"}$
$(H_{rock} := 1.6\ \text{m}) = \text{"Heigth from A (Toe) to Rock Layer"}$
$(WIDTH := 1\ \text{m}) = \text{"Define width of strips for Stability Analysis"}$

Define Array of Centers

$$CENS6 := \begin{bmatrix} 10 & 10 \\ 8.5 & 22 \\ 8.5 & 18 \\ 7 & 5 \\ 4 & 5 \\ 1.5 & 5 \\ 1 & 5 \\ 1 & 1.1 \\ 1 & 1 \\ 1 & 2 \\ 0.5 & 1 \\ 0 & 2 \\ 0 & 1 \\ -3 & 1 \\ -5 & 1 \\ -10 & 1 \\ -0.59 & 1 \\ -0.6 & 1 \end{bmatrix} \text{m}$$

1. Calculations

$AB' := \frac{H_{bund}}{\beta_{slope}} = 16\ \text{m}$ $OO' := AB' - 6\ \text{m} = 10\ \text{m}$

$AB := \sqrt{H_{bund}^{\ 2} + AB'^{\ 2}} = 17.8885\ \text{m}$

1.1 Define Slope Toe Cordinates

$A := [\ 1\ \ 1\ ]\ m$

Note: Pl do **not change cordinates of A,** as all cordinates read from **ACAD** are based on this defined value of A

$[\ 1\ m\ \ 1\ m\ ] = [\ 1\ \ 1\ ]\ m$

1.2 Define Trial Center

$Center := [\ 8.5\ \ 18\ ]\ m$

1.3 Radius of Circle from Center to touch the rock layer

$R := Center_2 - A_2 + H_{rock} = 18.6\ m$

Distance from A to B

$B := \left[\left(AB' + A_1 \right) \left(H_{bund} + A_2 \right) \right] = [\ 17\ \ 9\ ]\ m$

Slope of Line AB

$SL := \beta_{slope} = 0.5$

Intercept of Line AB using c = y - mx

$SP := A_{1\,2} - SL \cdot A_{1\,1} = 0.5\ m$

X cord beyond which both the inersecting pt is in hoz line

$\Delta x_{neg\_cen} := A_2 - H_{rock} = -0.6\ m$

Distance from Center to A $\quad L\_CA := \sqrt{\left(Center_1 - A_1 \right)^2 + \left(Center_2 - A_2 \right)^2} = 18.58\ m \qquad L\_CA > R = 0$

Distance from Center to B $\quad L\_CB := \sqrt{\left(Center_1 - B_1 \right)^2 + \left(Center_2 - B_2 \right)^2} = 12.38\ m \qquad L\_CB > R = 0$

2 Analysis

Program 1 : To find intersection points of Circle with slope profile

$$Find\_aa\_cc(Pt, cen\#) :=$$

$$rr := \frac{\left(cen\#_2 - A_2 \right) + H_{rock}}{m}$$

$$L\_ca := \sqrt{\left(\frac{cen\#_1 - A_1}{m} \right)^2 + \left(\frac{cen\#_2 - A_2}{m} \right)^2}$$

$$L\_cb := \sqrt{\left(\frac{cen\#_1 - B_1}{m} \right)^2 + \left(\frac{cen\#_2 - B_2}{m} \right)^2}$$

$$gs\# := \begin{cases} y - SL \cdot x - \frac{SP}{m} & \text{if } (L\_ca > rr) \wedge (Pt = A) \\ y - SL \cdot x - \frac{SP}{m} & \text{if } (L\_cb > rr) \wedge (Pt = B) \\ y - \frac{Pt_2}{m} & \text{otherwise} \end{cases}$$

$$f(x, y) := \left[\left(\left(x - \frac{cen\#_{1\,1}}{m} \right)^2 + \left(y - \frac{cen\#_{1\,2}}{m} \right)^2 - rr^2 \right) \right]_{gs\#}$$

$$ans := \begin{cases} roots\left(f(x, y), \begin{bmatrix} x \\ y \end{bmatrix}, \begin{bmatrix} \frac{-Pt_1}{m} \\ \frac{Pt_2}{m} \end{bmatrix} \right)^T m & \text{if } \left(cen\#_1 \le A_1 \right) \wedge (Pt = A) \\ roots\left(f(x, y), \begin{bmatrix} x \\ y \end{bmatrix}, \begin{bmatrix} \frac{Pt_1}{m} \\ \frac{Pt_2}{m} \end{bmatrix} \right)^T m & \text{otherwise} \end{cases}$$

$Center = [\ 8.5\ m\ \ 18\ m\ ]$

$L\_CB = 12.38\ m$

$R = 18.6\ m$

$L\_CA > R = 0$

$L\_CB > R = 0$

$Center_1 < A_1 = 0$

$Center_1 < 0 = 0$

Program 2 : To deal with -ve x cord of centers

$$Find\_Neg(Pt, cen) := \left| \begin{array}{l} rr := \left(cen_2 - A_2\right) + H_{rock} \\ \begin{cases} \begin{bmatrix} cen_1 - rr \\ cen_2 \end{bmatrix}^T & \text{if } \left(cen_1 \le \Delta x_{neg\_cen}\right) \wedge (Pt = A) \\ \begin{bmatrix} cen_1 + rr \\ cen_2 \end{bmatrix}^T & \text{otherwise} \end{cases} \end{array} \right.$$

Program 3 : To stack and array

$$STACK(M\#) := \left| \begin{array}{l} \text{for } j \in [1..\text{rows}(M\#)] \\ \quad \left| \begin{array}{l} \text{if } j = 1 \\ \quad bb := M\#_j \\ \text{else} \\ \quad bb := \text{stack}\left(bb, M\#_j\right) \end{array} \right. \\ bb \end{array} \right.$$

Program 4 :

$$Find\_Any(Pt, cen) := \left| ans := \begin{cases} Find\_Neg(Pt, cen) & \text{if } cen_1 \le -0.60\,\text{m} \\ Find\_aa\_cc(Pt, cen) & \text{otherwise} \end{cases} \right.$$

Program 25 : Sample program to find the Number of Slices in Crtical Failure Circle

$$Find\_Slices\_Tot(A, aa, B, cc, width) := \left| \begin{array}{l} slices\_AB := \begin{cases} \left\lceil \left| \frac{aa_1 - B_1}{width} \right| \right\rceil & \text{if } \left(aa_1 > A_1\right) \wedge \left(B_1 > aa_1\right) \\ \left\lceil \left| \frac{A_1 - B_1}{width} \right| \right\rceil & \text{if } \left(A_1 > aa_1\right) \wedge \left(cc_1 > B_1\right) \\ \left\lceil \left| \frac{A_1 - cc_1}{width} \right| \right\rceil & \text{otherwise} \end{cases} \\ slices\_HOZ1 := \begin{cases} 0 & \text{if } aa_1 > A_1 \\ \left\lceil \left| \frac{A_1 - aa_1}{width} \right| \right\rceil & \text{otherwise} \end{cases} \\ slices\_HOZ2 := \begin{cases} 0 & \text{if } cc_1 < B_1 \\ \left\lceil \left| \frac{B_1 - cc_1}{width} \right| \right\rceil & \text{otherwise} \end{cases} \\ slices\_Tot := slices\_AB + slices\_HOZ1 + slices\_HOZ2 \end{array} \right.$$

Program 5 : To analyze array of centers

$$Find\_All\_AA\_CC(M, width) := \left| \begin{array}{l} j := [1..\text{rows}(M)] \\ rr_j := \left(M_j[1..2]_2 - A_2\right) + H_{rock} \\ Z1_j := Find\_Any\left(A, M_j[1..2]\right) \\ Z2_j := Find\_Any\left(B, M_j[1..2]\right) \\ Z3_j := Find\_Slices\_Tot\left(A, Z1_j, B, Z2_j, width\right) \\ Z := \text{augment}(Z1, Z2, Z3) \\ Z := \left[[M]\ rr\ Z1\ Z2\ Z3 \right] \end{array} \right.$$

3. Results for the TRIAL CENTER

$Center = [8.5\ \ 18]\,\text{m}$ $\quad R = 18.6\,\text{m}$ $\quad Center_1 \le -0.60\,\text{m} = 0$

$AA := Find\_Any(A, Center) = [0.95\,\text{m}\ \ 1\,\text{m}]$ $\quad CC := Find\_Any(B, Center) = [24.78\,\text{m}\ \ 9\,\text{m}]$

$Find\_Slices\_Tot(A, AA, B, CC, WIDTH) = 25$

4. Results for the ARRAY of CENTER

4.1 Define Header for the output Table

$HD := \text{augment}(\text{"Center X"}, \text{"Center Y"}, \text{"Radius"}, \text{"AA\_X"}, \text{"AA\_Y"}, \text{"CC\_X"}, \text{"CC\_Y"}, \text{"Slices"})$

Apply Program 5

$$Res1 := Find\_All\_AA\_CC(CENS6, WIDTH) = \begin{bmatrix} \begin{bmatrix} 10\text{ m} & 10\text{ m} \\ 8.5\text{ m} & 22\text{ m} \\ 8.5\text{ m} & 18\text{ m} \\ 7\text{ m} & 5\text{ m} \\ 4\text{ m} & 5\text{ m} \\ 1.5\text{ m} & 5\text{ m} \\ 1\text{ m} & 5\text{ m} \\ 1\text{ m} & 1.1\text{ m} \\ 1\text{ m} & 1\text{ m} \\ 1\text{ m} & 2\text{ m} \\ 0.5\text{ m} & 1\text{ m} \\ 0 & 2\text{ m} \\ 0 & 1\text{ m} \\ -3\text{ m} & 1\text{ m} \\ -5\text{ m} & 1\text{ m} \\ -10\text{ m} & 1\text{ m} \\ -0.59\text{ m} & 1\text{ m} \\ -0.6\text{ m} & 1\text{ m} \end{bmatrix} \begin{bmatrix} 10.6\text{ m} \\ 22.6\text{ m} \\ 18.6\text{ m} \\ 5.6\text{ m} \\ 5.6\text{ m} \\ 5.6\text{ m} \\ 5.6\text{ m} \\ 1.7\text{ m} \\ 1.6\text{ m} \\ 2.6\text{ m} \\ 1.6\text{ m} \\ 2.6\text{ m} \\ 1.6\text{ m} \\ 1.6\text{ m} \\ 1.6\text{ m} \\ 1.6\text{ m} \\ 1.6\text{ m} \end{bmatrix} \begin{bmatrix} [3.03\text{ m}\ 2.01\text{ m}] \\ [0.15\text{ m}\ 1\text{ m}] \\ [0.95\text{ m}\ 1\text{ m}] \\ [2.46\text{ m}\ 1.73\text{ m}] \\ [0.08\text{ m}\ 1\text{ m}] \\ [-2.42\text{ m}\ 1\text{ m}] \\ [-2.92\text{ m}\ 1\text{ m}] \\ [-0.7\text{ m}\ 1\text{ m}] \\ [-0.6\text{ m}\ 1\text{ m}] \\ [-1.4\text{ m}\ 1\text{ m}] \\ [-1.1\text{ m}\ 1\text{ m}] \\ [-2.4\text{ m}\ 1\text{ m}] \\ [-1.6\text{ m}\ 1\text{ m}] \\ [-4.6\text{ m}\ 1\text{ m}] \\ [-6.6\text{ m}\ 1\text{ m}] \\ [-11.6\text{ m}\ 1\text{ m}] \\ [-2.19\text{ m}\ 1\text{ m}] \\ [-2.2\text{ m}\ 1\text{ m}] \end{bmatrix} \begin{bmatrix} [20.55\text{ m}\ 9\text{ m}] \\ [26.99\text{ m}\ 9\text{ m}] \\ [24.78\text{ m}\ 9\text{ m}] \\ [12.34\text{ m}\ 6.67\text{ m}] \\ [9.59\text{ m}\ 5.3\text{ m}] \\ [7.01\text{ m}\ 4.01\text{ m}] \\ [6.45\text{ m}\ 3.73\text{ m}] \\ [2.56\text{ m}\ 1.78\text{ m}] \\ [2.43\text{ m}\ 1.72\text{ m}] \\ [3.58\text{ m}\ 2.29\text{ m}] \\ [2.02\text{ m}\ 1.51\text{ m}] \\ [2.59\text{ m}\ 1.8\text{ m}] \\ [1.57\text{ m}\ 1.29\text{ m}] \\ [-1.4\text{ m}\ 1\text{ m}] \\ [-3.4\text{ m}\ 1\text{ m}] \\ [-8.4\text{ m}\ 1\text{ m}] \\ [1.01\text{ m}\ 1\text{ m}] \\ [1\text{ m}\ 1\text{ m}] \end{bmatrix} \begin{bmatrix} 18 \\ 27 \\ 25 \\ 15 \\ 10 \\ 11 \\ 10 \\ 4 \\ 4 \\ 6 \\ 5 \\ 6 \\ 4 \\ 9 \\ 13 \\ 23 \\ 5 \\ 4 \end{bmatrix} \end{bmatrix}$$

$$Res2 := \overrightarrow{STACK(Res1)} = \begin{bmatrix} \begin{bmatrix} 10\text{ m} & 10\text{ m} \\ 8.5\text{ m} & 22\text{ m} \\ 8.5\text{ m} & 18\text{ m} \\ 7\text{ m} & 5\text{ m} \\ 4\text{ m} & 5\text{ m} \\ 1.5\text{ m} & 5\text{ m} \\ 1\text{ m} & 5\text{ m} \\ 1\text{ m} & 1.1\text{ m} \\ 1\text{ m} & 1\text{ m} \\ 1\text{ m} & 2\text{ m} \\ 0.5\text{ m} & 1\text{ m} \\ 0 & 2\text{ m} \\ 0 & 1\text{ m} \\ -3\text{ m} & 1\text{ m} \\ -5\text{ m} & 1\text{ m} \\ -10\text{ m} & 1\text{ m} \\ -0.59\text{ m} & 1\text{ m} \\ -0.6\text{ m} & 1\text{ m} \end{bmatrix} \begin{bmatrix} 10.6\text{ m} \\ 22.6\text{ m} \\ 18.6\text{ m} \\ 5.6\text{ m} \\ 5.6\text{ m} \\ 5.6\text{ m} \\ 5.6\text{ m} \\ 1.7\text{ m} \\ 1.6\text{ m} \\ 2.6\text{ m} \\ 1.6\text{ m} \\ 2.6\text{ m} \\ 1.6\text{ m} \\ 1.6\text{ m} \\ 1.6\text{ m} \\ 1.6\text{ m} \\ 1.6\text{ m} \end{bmatrix} \begin{bmatrix} 3.03\text{ m} & 2.01\text{ m} \\ 0.15\text{ m} & 1\text{ m} \\ 0.95\text{ m} & 1\text{ m} \\ 2.46\text{ m} & 1.73\text{ m} \\ 0.08\text{ m} & 1\text{ m} \\ -2.42\text{ m} & 1\text{ m} \\ -2.92\text{ m} & 1\text{ m} \\ -0.7\text{ m} & 1\text{ m} \\ -0.6\text{ m} & 1\text{ m} \\ -1.4\text{ m} & 1\text{ m} \\ -1.1\text{ m} & 1\text{ m} \\ -2.4\text{ m} & 1\text{ m} \\ -1.6\text{ m} & 1\text{ m} \\ -4.6\text{ m} & 1\text{ m} \\ -6.6\text{ m} & 1\text{ m} \\ -11.6\text{ m} & 1\text{ m} \\ -2.19\text{ m} & 1\text{ m} \\ -2.2\text{ m} & 1\text{ m} \end{bmatrix} \begin{bmatrix} 20.55\text{ m} & 9\text{ m} \\ 26.99\text{ m} & 9\text{ m} \\ 24.78\text{ m} & 9\text{ m} \\ 12.34\text{ m} & 6.67\text{ m} \\ 9.59\text{ m} & 5.3\text{ m} \\ 7.01\text{ m} & 4.01\text{ m} \\ 6.45\text{ m} & 3.73\text{ m} \\ 2.56\text{ m} & 1.78\text{ m} \\ 2.43\text{ m} & 1.72\text{ m} \\ 3.58\text{ m} & 2.29\text{ m} \\ 2.02\text{ m} & 1.51\text{ m} \\ 2.59\text{ m} & 1.8\text{ m} \\ 1.57\text{ m} & 1.29\text{ m} \\ -1.4\text{ m} & 1\text{ m} \\ -3.4\text{ m} & 1\text{ m} \\ -8.4\text{ m} & 1\text{ m} \\ 1.01\text{ m} & 1\text{ m} \\ 1\text{ m} & 1\text{ m} \end{bmatrix} \begin{bmatrix} 18 \\ 27 \\ 25 \\ 15 \\ 10 \\ 11 \\ 10 \\ 4 \\ 4 \\ 6 \\ 5 \\ 6 \\ 4 \\ 9 \\ 13 \\ 23 \\ 5 \\ 4 \end{bmatrix} \end{bmatrix}$$

$$R3 := \frac{\text{augment}\left(Res2_{1\,1}, Res2_{1\,2}, Res2_{1\,3}, Res2_{1\,4}, Res2_{1\,5}\text{ m}\right)}{\text{m}}$$

Center X	Center Y	Radius	AA_X	AA_Y	CC_X	CC_Y	Slices
10	10	10.6	3.03	2.01	20.55	9	18
8.5	22	22.6	0.15	1	26.99	9	27
8.5	18	18.6	0.95	1	24.78	9	25
7	5	5.6	2.46	1.73	12.34	6.67	15
4	5	5.6	0.08	1	9.59	5.3	10
1.5	5	5.6	-2.42	1	7.01	4.01	11
1	5	5.6	-2.92	1	6.45	3.73	10
1	1.1	1.7	-0.7	1	2.56	1.78	4
1	1	1.6	-0.6	1	2.43	1.72	4
1	2	2.6	-1.4	1	3.58	2.29	6
0.5	1	1.6	-1.1	1	2.02	1.51	5
0	2	2.6	-2.4	1	2.59	1.8	6
0	1	1.6	-1.6	1	1.57	1.29	4
-3	1	1.6	-4.6	1	-1.4	1	9
-5	1	1.6	-6.6	1	-3.4	1	13
-10	1	1.6	-11.6	1	-8.4	1	23
-0.59	1	1.6	-2.19	1	1.01	1	5
-0.6	1	1.6	-2.2	1	1	1	4

--------Verified with ACAD ----------

X = 3.03 Y = 2.01	X = 20.55 Y = 9.00	✓
X = 0.15 Y = 1.00	X = 26.99 Y = 9.00	✓
X = 0.95 Y = 1.00	X = 24.78 Y = 9.00	✓
X = 2.46 Y = 1.73	X = 12.34 Y = 6.67	✓
X = 0.08 Y = 1.00	X = 9.59 Y = 5.30	✓
X = -2.42 Y = 1.00	X = 7.01 Y = 4.01	✓
X = -2.92 Y = 1.00	X = 6.45 Y = 3.73	✓
X = -0.70 Y = 1.00	X = 2.56 Y = 1.78	✓
X = -0.60 Y = 1.00	X = 2.43 Y = 1.72	✓
X = -1.40 Y = 1.00	X = 3.58 Y = 2.29	✓
X = -1.10 Y = 1.00	X = 2.02 Y = 1.51	✓
X = -2.40 Y = 1.00	X = 2.59 Y = 1.80	✓
X = -1.60 Y = 1.00	X = 1.57 Y = 1.29	✓
X = -4.60 Y = 1.00	X = -1.4 Y = 1.00	✓
X = -6.60 Y = 1.00	X = -3.4 Y = 1.00	✓
X = -11.60 Y = 1.00	X = -8.4 Y = 1.00	✓
X = -2.19 Y = 1.005	X = 1.10 Y = 1.05	✓
X = -2.20 Y = 1.00	X = 1.00 Y = 1.00	✓

$$\text{time}(0) - t_0 = 0.1\text{ s}$$